GEODESIC DOME

Structural Analysis

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Revision History

Number	Date	Comment	Author
A	12 Jan 2005	Original	Ben Lewis
B	5 April 2005	Added modified node calculations	Ben Lewis

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INTRODUCTION

The purpose of this report is to analyse the 9 m geodesic dome manufactured by Dome Dimensions to determine the safe working load of the structure. It has been assessed in accordance with AS4100 for steel structures and AS1170 for loading conditions.

Figure 1: Dome Dimensions geodesic dome

Figure 2: Dome Dimensions geodesic hub joint

DOME CHARACTERISTICS

The geodesic characteristics of the Dome Dimensions 9 m dome are listed in Table 1.

Table 1: Dome Dimensions 9 m dome characteristics

Item	*Value*
Dome type	Icosahedron dome (modified base)
Base diameter	8.86 m
Spherical diameter	9.00 m
Height	3.704 m
Frequency	3
Class	1
Eccentricity	1.00
Horizontal projected area	61.62 m^2
Vertical projected area	24.68 m^2
Spherical angle	159.61°
Spherical dome surface area	104.72 m^2
Spherical dome volume	140.72 m^3
Number of hubs	46
Number of base boundary points	15
Number of struts	120
Total length of struts	212.47 m
Longest strut	1.90 m
Shortest strut	1.55 m
Number of panels	75
Total area of panels	100.98 m^2
Largest panel	1.51 m^2
Smallest panel	1.14 m^2

The materials used in the construction of the Dome Dimensions 9 m dome are listed in Table 2.

Table 2: Construction materials

Item	*Value*
Strut	Tubeline R.H.S. C350L0 75 x 50 x 2.0 mm
Node ring	Tubeline C.H.S. C250L0 88.9 x 4.0 mm
Profile plates	Mild Steel 5.0 mm
Fasteners	M10, Grade 8.8, button head cap screw

The mass of the geodesic dome has been calculated using the items listed in Table 3.

Table 3: Dome mass

Item	Unit Mass	Qty	Total [kg]
RHS struts	3.72 kg/m	212 – (0.07 x 2 x 120) = 195 m	726
CHS Rings	8.38 kg/m	0.045 x 46 = 2.07 m	17
Profile Clamps	0.180 kg	4 x 2 x 120 = 960	173
Locking washers	0.110 kg	46 x 2 = 92	10
Fasteners	0.010 kg	120 x 3 = 360	4
Tarp	1.0 kg/m^2	100 m^2	100
Total			1,030

ASSUMPTIONS

The following assumptions have been made in this analysis.

- The canvas canopy has been represented as a series of vertical loads applied at each node. In reality there will be an additional horizontal component as the tarp tension acts in a direction normal to the dome surface.
- Wind loading is assumed to act in a direction parallel to the ground plane on the vertical projected area of the dome.
- It is assumed that working loads are suspended from the geometric centre of a node. In reality any mass suspended from a node will be attached via a bolt thus offsetting the load from the node centre. This will create a small torque component not accounted for in this analysis.
- The dome is assumed to be mounted on flat level ground
- It is assumed that each member on the base of the dome is fixed to the ground via a stake or similar means.
- The safe working load of this structure has been assessed against a serviceability state criterion and an ultimate state criterion. The serviceability state criterion is designed to ensure the structure does not experience any permanent deformation of parts under normal operating conditions. The ultimate state criterion is designed to ensure the structure does not collapse under the worst-case conditions, although some members may experience local yielding.

REFERENCED STANDARDS

AS 1170: 2002	Structural Design Actions
AS 4100: 1998	Steel Structures
AISC - ASD	Allowable Stress Design - Buckling of Compact Rolled Shapes

FAILURE MODES

This section examines the most likely cause of structural failure. The weakest member in the structure is determined by comparing the magnitude of the force for each failure mode. Table 4 lists each failure mode examined here with the corresponding strut force. It can be seen that the node is the weakest component in the structure and is expected to yield at approximately 6.0 kN of applied load from the strut.

Table 4: Failure mode with estimated force

	Compression Members [kN]	Tension Members [kN]
Buckling force about pinned axis	55	N/A
Buckling force about fixed axis	47	N/A
Bolt-strut normal force	14	14
Strut shear force	N/A	42
Bolt shear force	55	55
Ring force	6.0	6.0

Strut Buckling

The buckling strength of the longest member has been assessed in accordance with the AISC (American Institute of Steel Construction) standard using the allowable stress design (ASD) method for compact rolled shapes.

The characteristics of each of the struts are shown in Table 5.

Table 5: Strut characteristics

Item	Value
Cross sectional area	384×10^{-6} m^2
Length A	1.569 m
Length B	1.816 m
Length C	1.856 m
Dead length	0.015 m
Max length pin to pin	1.826 m
Strength limit (sigma x A)	134 kN

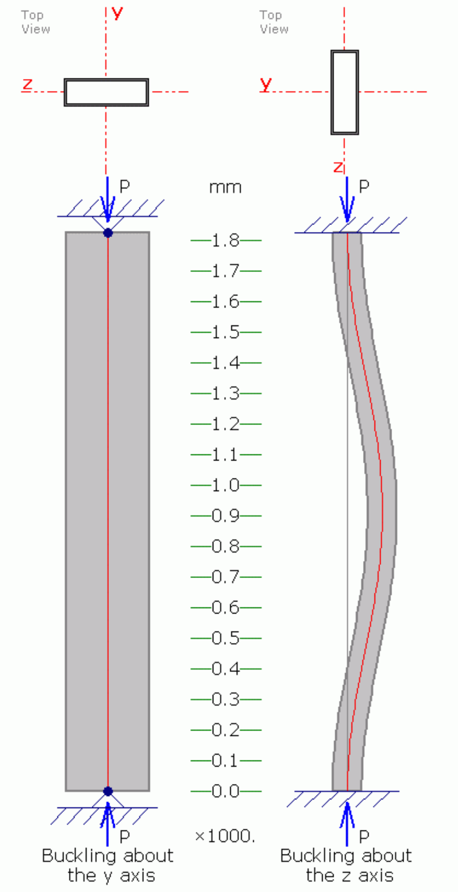


Figure 3: Strut buckling analysis

Table 6 shows the calculations used to determine the buckling strength of the longest member in both the pined and fixed axes. The strut is weakest in buckling about its fixed (Z) axis. Buckling is expected to occur at 47.2 kN.

Table 6: Strut buckling calculations

Item	Formula	Buckling about pinned (Y) axis	Buckling about fixed (Z) axis
Mass moment of inertia [mm^4]	$I = \frac{bh^3}{12} - \frac{b_i h_i^3}{12}$	252×10^3	42.9×10^3
Radius of gyration [mm]	$r = \sqrt{\frac{I}{A}}$	25.6	10.6
Slenderness ratio	$\frac{KL}{r}$	71.2	86.4
Critical slenderness ratio	$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$	106	106
Buckling mode	$\frac{KL}{r} \le C_c$ -> Inelastic $\frac{KL}{r} > C_c$ -> Elastic	Inelastic	Inelastic
Inelastic bucking stress [MPa]	$\sigma_a = \frac{\left[1 - \frac{(KL/r)}{2C_c^2}\right]\sigma_y}{\frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^3}}$	144	123
Nominal buckling force [kN]	$F_a = A\sigma_a$	55.4	47.2
Design buckling force [kN]	S.F = 1.67	33.2	28.3

Bolt Failure

Strut fasteners are M10, grade 8.8 button head cap screws. Table 7 shows the calculations used to determine the shear capacity of these bolts. These calculations have been completed in accordance with AS4100:1998. It can be seen that the nominal shear capacity of these bolts is 60 kN and the design capacity is 48 kN

Table 7: Bolt shear capacity

Item	Formula	Value
Bolt minimum tensile strength (see Table 9.3.1)	f_{uf}	830 MPa
Bolted lap connection, length reduction factor (see Table 9.3.2.1)	k_r	1.0
Number of shear planes with threads intercepting the shear plane	n_n	2
Minor diameter area of the bolt (see AS 1275 Table 3.3)	A_c	58
Number of shear planes without threads intercepting the shear plane	n_x	0
Nominal plain shank area of the bolt	A_o	78.5
Capacity factor (see Table 3.4)	ϕ	0.8
Bolt nominal shear capacity	$V_f = 0.62 f_{uf} k_r (n_n A_c + n_x A_o)$	60 kN
Bolt design shear	$V_f^* \leq \phi V_f$	48 kN

The strut may fail in the region around the bolt due to localised stresses. Two failure modes have been investigated here. The strut may yield due to an excessive compressive force on the bolt-bearing surface; it may also fail in shear as the bolt tries to pull through the strut material holding it captive. The calculations in Table 8 have been used to determine the approximate forces at which these failures will occur. It can be seen that the strut material will begin to yield in compression around the bolt at approximately 14 kN. It should be noted that yielding in this region is not likely to result in a catastrophic failure of the entire structure it will simply cause an elongation of the bolt hole. This would only be a problem if this loading were experienced repetitively over an ongoing period of time.

Table 8: Local strut failure around the bolted connection

Item	Formula	Value
Bolt bearing normal force	$F_{bb} = D_{bolt} (b - b_i) \sigma_y$	14.0 kN
Strut shear force	$F_{ss} = 2L_{dead} (b - b_i) \sigma_y$	42.0 kN

Node Ring Deformation

FEA analysis identified the ring to be the weakest component in the hub. The load carrying capacity of this ring was investigated using three methods, hand calculations, FEA analysis and experimentation. Table 4 below shows the results obtained from each method. It can be seen that no hand calculations or experimental data was available for the node with locking washers but it can be seen from the results of the analysis without locking rings that the FEA method was conservative. A nominal value of 6.0 kN was selected as the maximum allowable strut force.

Figure 4: Node ring deformation estimates using various methods

	Strut Force [kN]	
Method	W/O Locking Washers	With Locking Washers
Hand calculation	2.2	NA
FEA analysis	2.0	6.0
Experimentation	4.0	NA

Hand Calculations

Basic formulas for curved beams indicate the load carrying capacity of the ring is 2.2 kN applied at two points on opposing sides of the ring, as shown in the left hand example of Figure 5. The majority of nodes in the dome are subject to combined loads of compression and tension acting at right angles to each other as shown in the right hand example of Figure 5. The nominal force in each strut to cause deformation of the ring is therefore approximately 1.1 kN, or half that determined with curved beam theory.

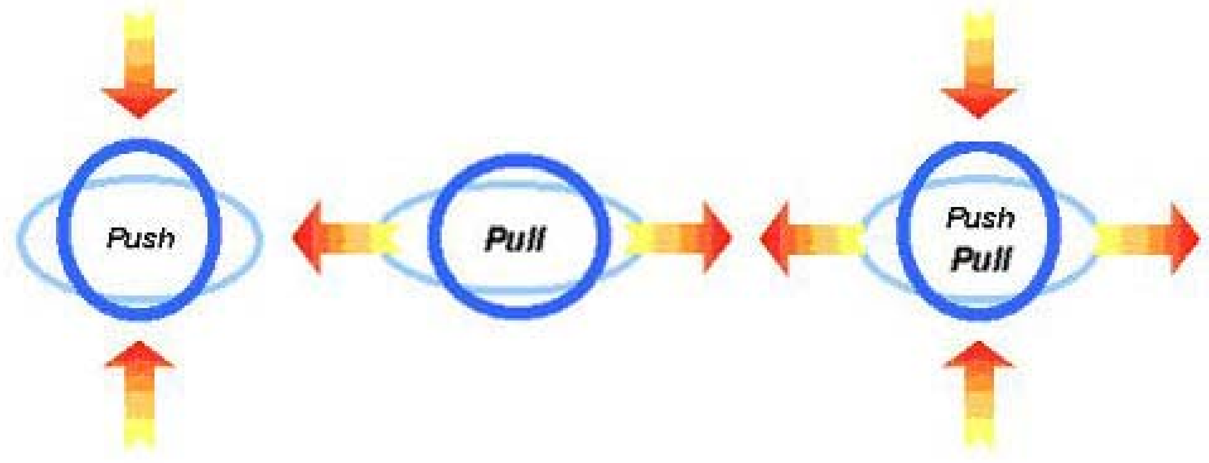


Figure 5: Combined effect of tension and compression on hub

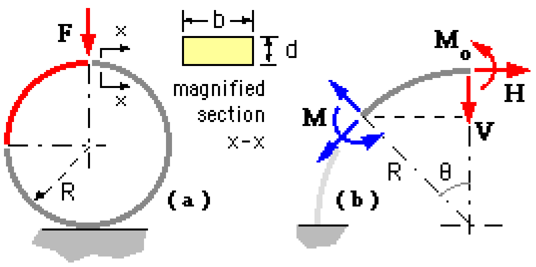


Table 9: Node ring calculations

Item	Formula	Value
Ring force	$F = \frac{\pi b d^2 \sigma}{6R}$	2.2 kN
Diametral deflection, coaxial with load	$\delta_V = \left(\frac{\pi}{4} - \frac{2}{\pi}\right)\frac{FR^3}{EI}$	0.53 mm
Diametral deflection, transverse to load	$\delta_H = \left(\frac{2}{\pi} - \frac{1}{2}\right)\frac{FR^3}{EI}$	0.48 mm

FEA Analysis

To analyses the geodesic structure a 3D geodesic dome was modelled using simulated beams. Beam features simulated include material type, cross-section, orientation and end releases. Three nodes were modelled in the dome to assess what stresses were induced in the ring under various loading scenarios. The location of the three nodes is shown in Figure 6. The top centre node was chosen because it is unique in that all attached members are in equally spaced and in compression. The two side nodes where chosen at the bottom of the structure because they carry the self weight of the structure, one of the nodes has five struts while the other has six.

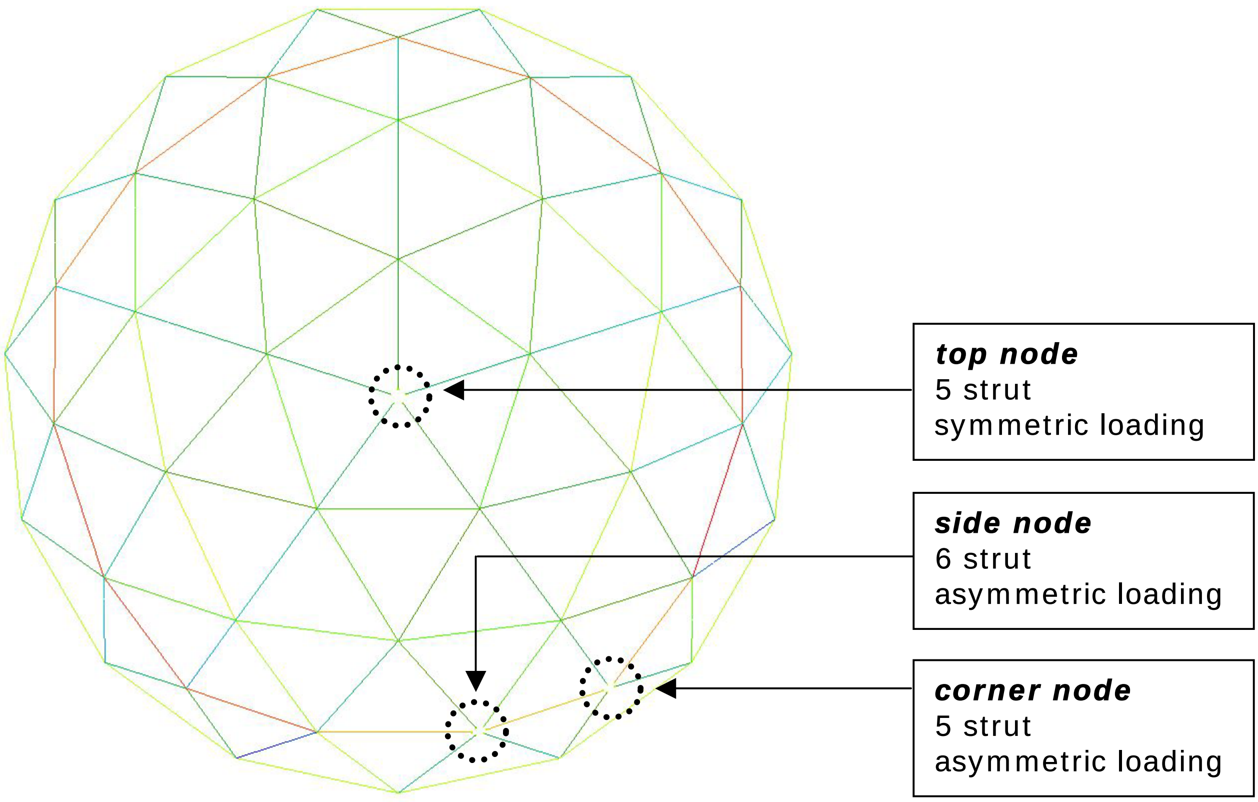


Figure 6: Location of nodes analysed on the structure

The dome model was loaded with masses at each node until the stress in the rings was in the vicinity of 210 to 240 MPa. The left hand column of Figure 7 shows a force of 2.0 kN in the struts was required to cause this level of stress. The addition of locking washers to the model raised the allowable force to between 6.0 and 12.0 kN depending on the node. This illustrates the importance of using the locking washers and also suggests that the ring will benefit from a permanent gusset welded to its centre. A nominal value of 6.0 kN was selected as the strut force maximum.

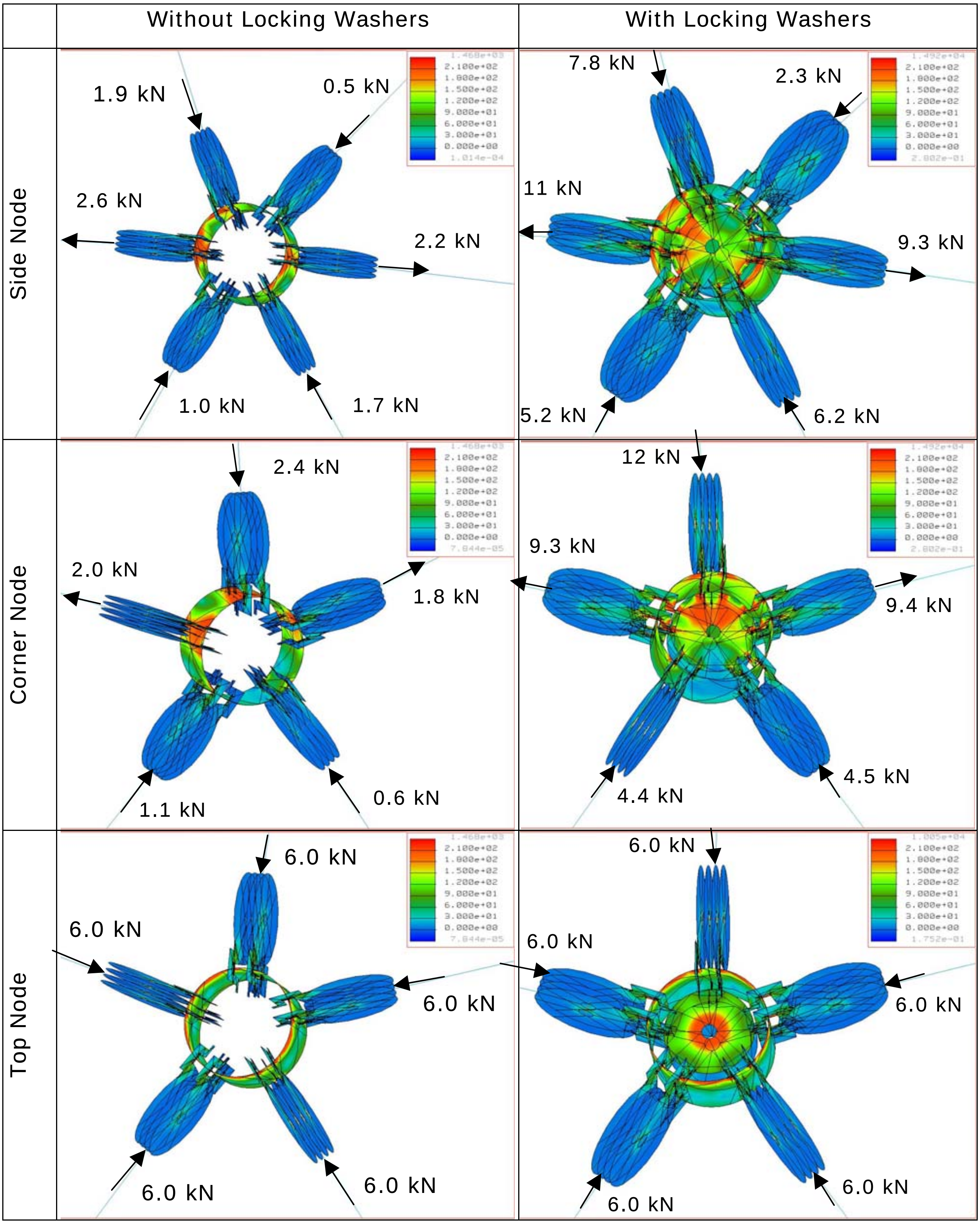


Figure 7: Node analysis

Experimentation

A node ring was compressed between two flat surfaces to obtain a force deflection curve, the results can be seen in Figure 8. It can be seen that the ring yielded at 4.0 kN and its ultimate strength was in excess of 6.0 kN.

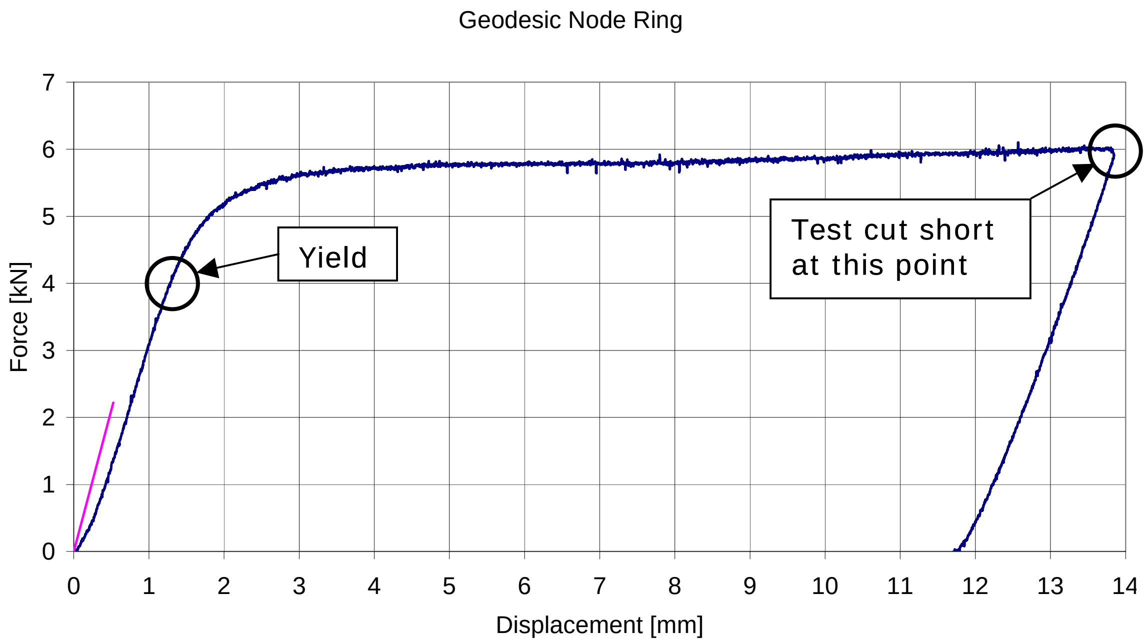


Figure 8: Node ring force - deflection curve

LOADS

This section determines the maximum allowable loads on the dome so as not to exceed the allowable strut for of 6.0 kN determined in the previous section. Loads investigated are wind loads and dead loads.

Wind Load

Figure 9 below shows the various wind regions throughout Australia. Region A was selected as the most appropriate for the dome. This means it can be erected anywhere in Australia, excluding cyclone affected coastal areas during cyclonic conditions (Regions B, C and D of AS1170.2: 2002).

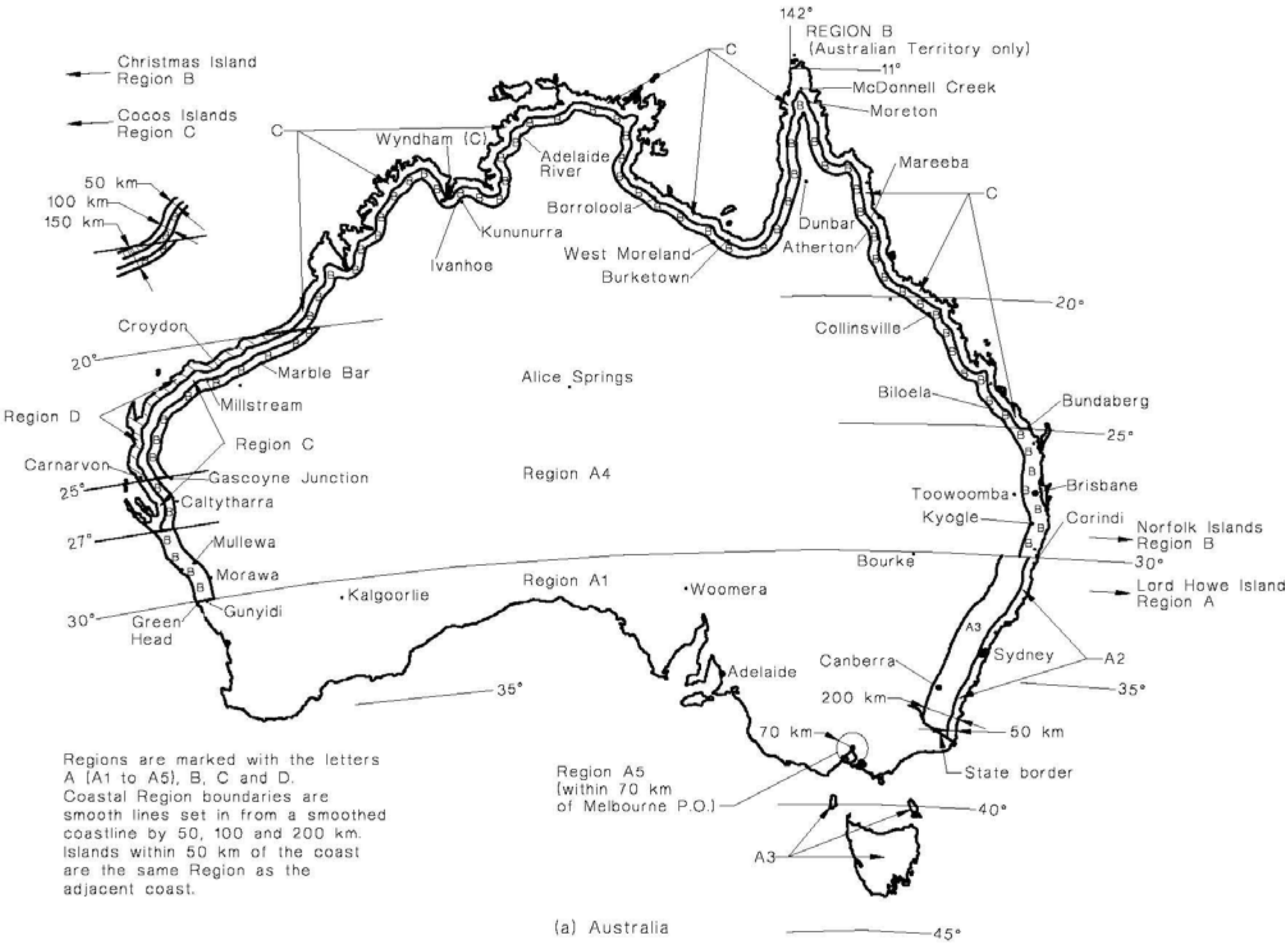


Figure 9: Wind speed regions

Table 10 shows the calculations used to determine wind loading in accordance with AS1170.2: 2002. It can be seen that for the serviceability limit state 550 N of compressive loading and 410 N of tensile loading must be allowed for. While for the ultimate limit state criteria 910 N of compressive loading and 680 N of tensile loading must be allowed for.

Table 10: Serviceability state site wind speed calculation

Item	Formula	Serviceability Limit State	Ultimate Limit State
Wind region	See Figure 9	A	A
Average recurrence interval (AS1170.2: 2002 Clause 3.2)	R	100 years	100 years
Regional wind speed (3 s gust) (AS1170.2: 2002 Clause 3.2)	$V_R = 67 - 41R^{-0.1}$	32 m/s	41 m/s
Wind directional multiplier (AS1170.2: 2002 Clause 3.3)	M_d	1.0	1.0
Terrain/height multiplier (AS1170.2: 2002 Clause 4.2)	$M_{z,cat}$	1.05 (Category 1, 5 m)	1.05 (Category 1, 5 m)
Shielding multiplier (AS1170.2: 2002 Clause 4.3)	M_s	1.0	1.0
Topographic multiplier (AS1170.2: 2002 Clause 4.4)	M_t	1.0 ($H/[2L_u] < 0.05$)	1.0 ($H/[2L_u] < 0.05$)
Site wind speed (AS1170.2: 2002 Clause 2.2)	$V_{sit,\beta} = V_R M_d \left(M_{z,cat} M_s M_t\right)$	34	43
Max compressive force (FEA analysis)	See Figure 10	550 N	910 N
Max tensile force (FEA analysis)	See Figure 11	410 N	680 N

Figure 10 shows the forces induced in each member under the serviceability limit state criteria in region A wind conditions.

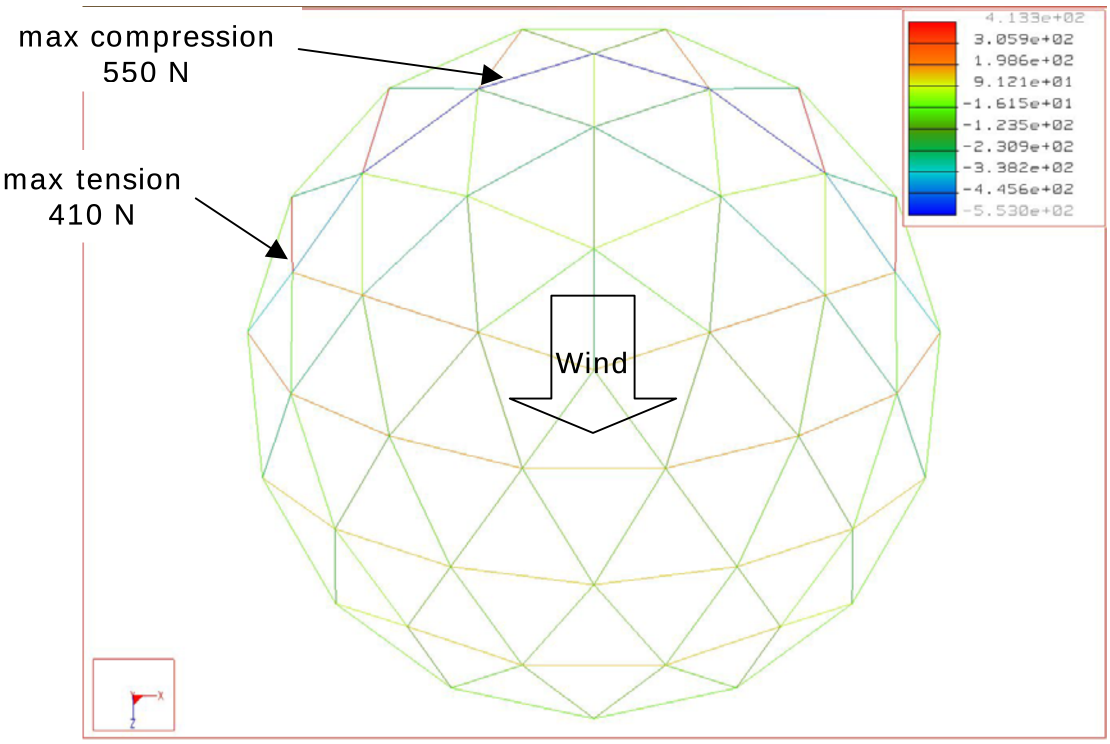


Figure 10: Beam forces under serviceability limit state

Figure 11 shows the forces induced in each member under the ultimate limit state criteria in region A wind conditions.

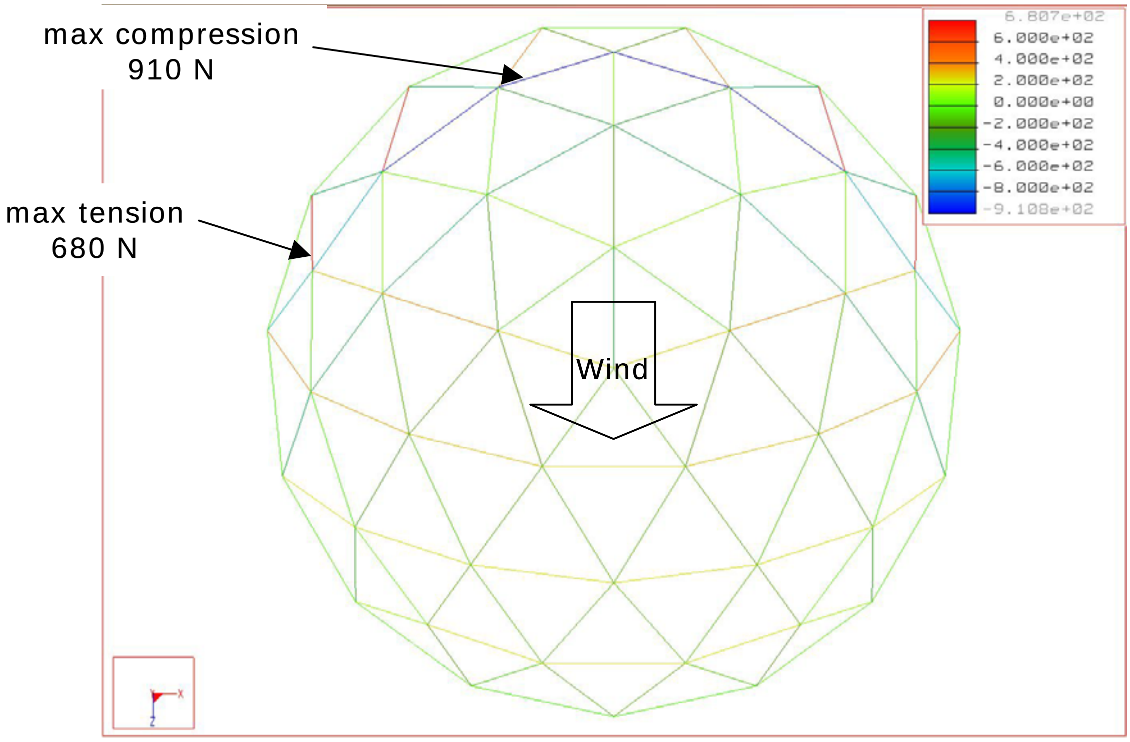


Figure 11: Beam forces under ultimate limit state

Dead Load

Figure 12 shows the forces induced in each member due to the structures own self weight.

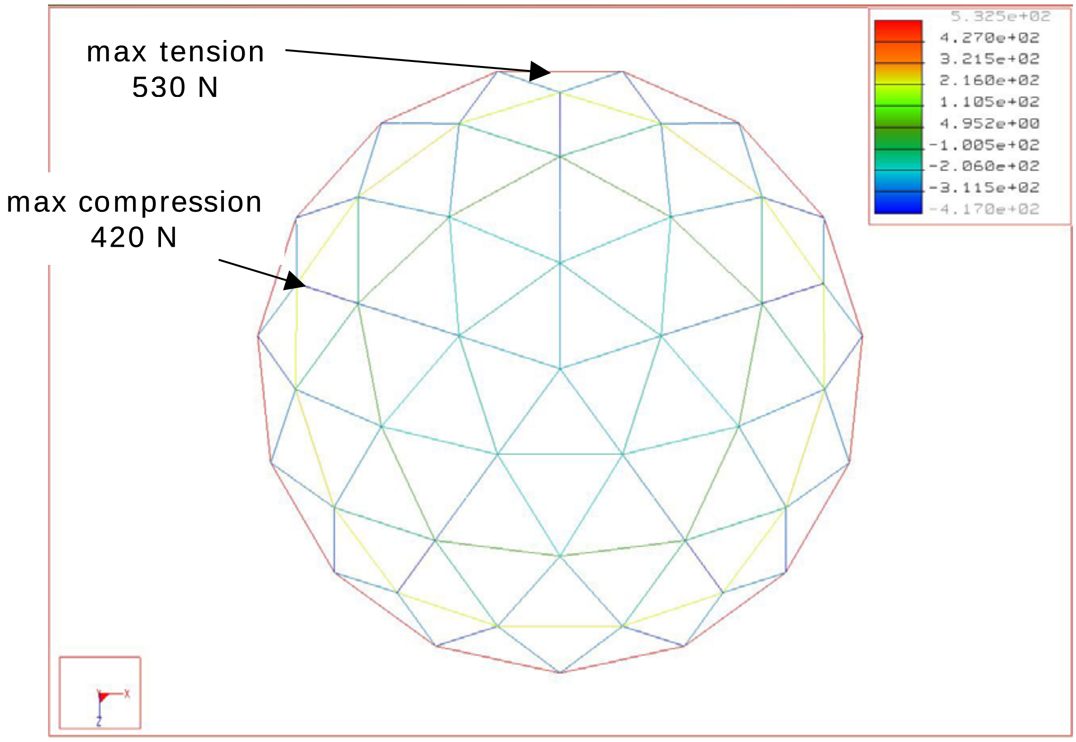


Figure 12: Beam forces due to structure dead weight

Combined Loading

Table 11 shows the calculations used to determine the available strut capacity of the structure. It can be seen that the dome members have 5.1 kN of capacity to carry the safe working load.

Table 11: Structure loads

	Serviceability [kN]	Ultimate [kN]
Maximum allowable	6.0	20.0
Dead	-0.5 x 0.8 = -0.4	-0.5
Wind	-0.5	-0.9
Available Strut capacity	5.1	18.6

Safe Working Load

Table 12 summarises the safe working load of the structure so as not to exceed the capacity of the members determined in the previous section. It can be seen that 200 kg can be suspended from all nodes or 400 kg can be suspended from a single node.

Table 12: Safe working loads

Nodes	Safe Working Load [kg]
All	200
One	400

Figure 13 shows the maximum tension and compression force in the dome when 2000 N is suspended from each node. It can be seen that the allowable force of 5.1 kN is not exceeded.

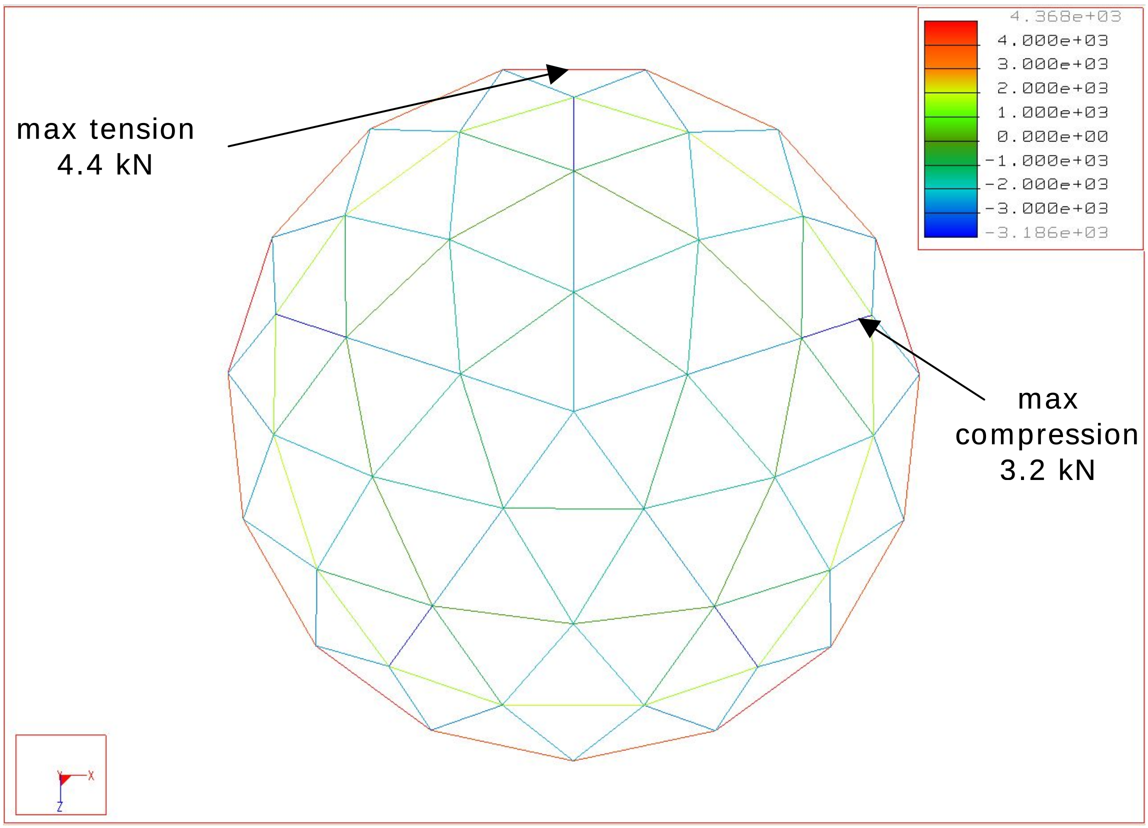


Figure 13: 2,000 N suspended from all nodes

Figure 14 to Figure 19 shows the affect on the structure of suspending a mass of 400 kg from a single node. It can be seen that the allowable force of 5.1 kN is equalled in the case of node 1 and not exceeded in all other cases.

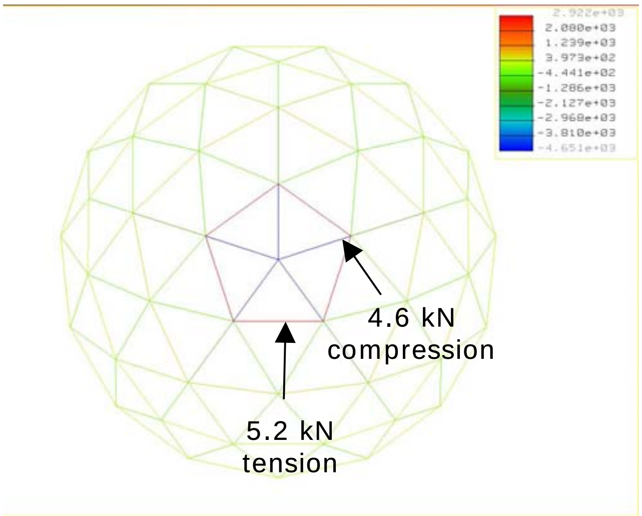


Figure 14: 4,000 N suspended from node 1

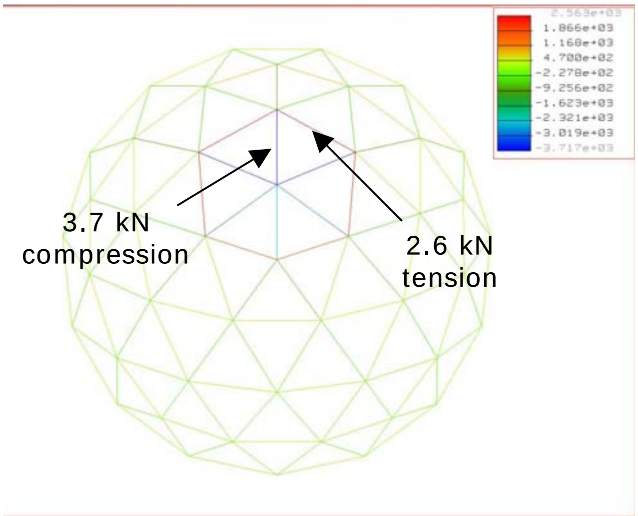


Figure 15: 4000 N suspended from node2

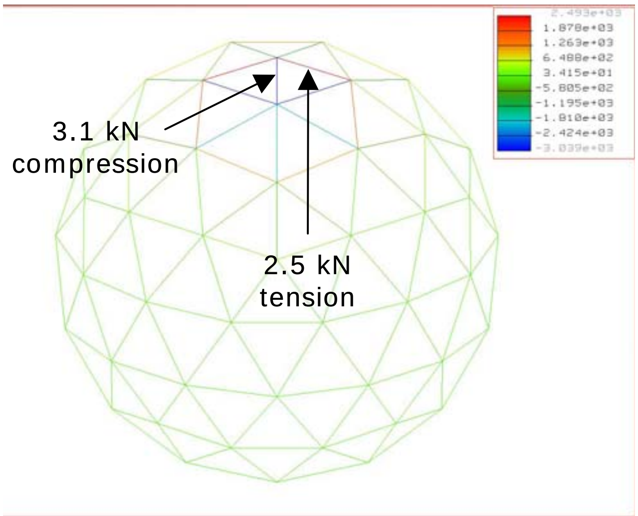


Figure 16: 4,000 N suspended from node 4

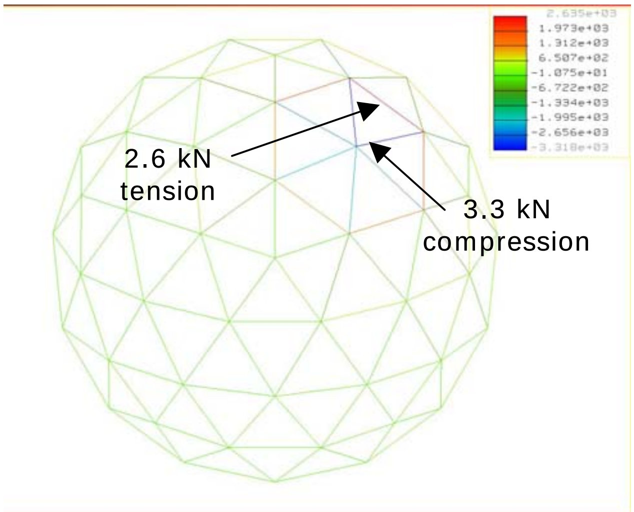


Figure 17: 4,000 N suspended from node 5

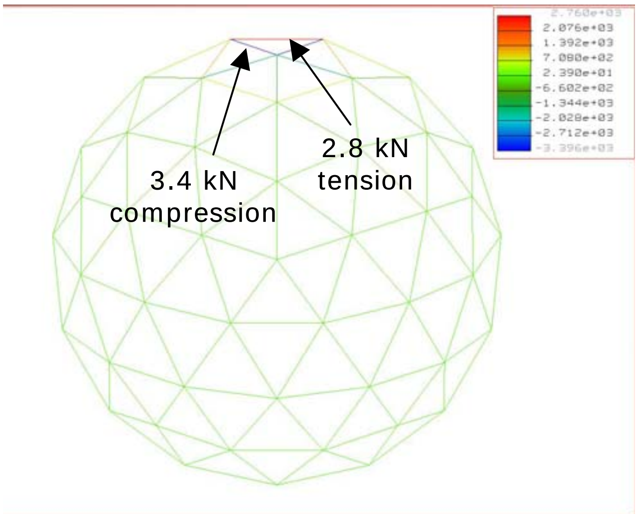


Figure 18: 4,000 N suspended from node 7

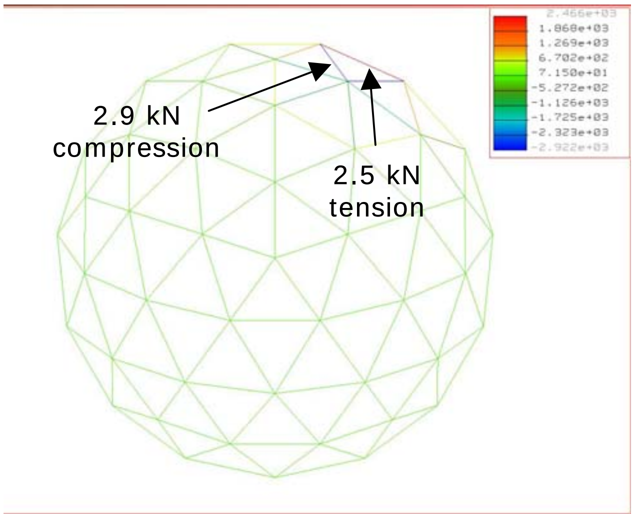
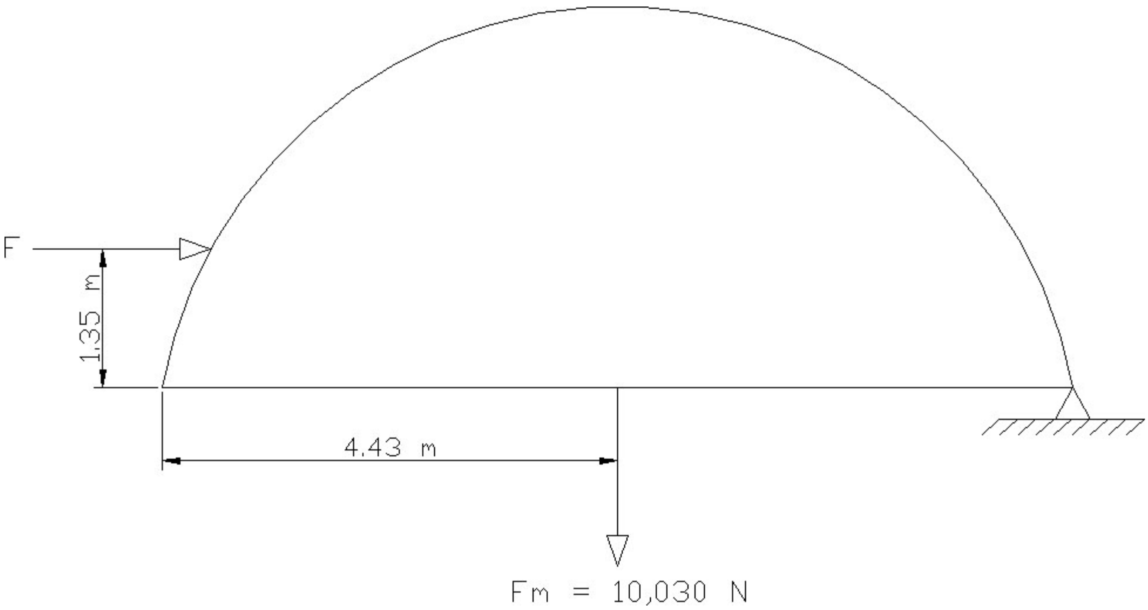


Figure 19: 4,000 N suspended from node 8

Overturning

To determine if overturning is likely to be a problem moments are taken about one edge of the dome. The overturing force is found to be 35.6 kN



The wind velocity required to create a force of this magnitude is approximately 100 m/s which is well below the 30 to 40 m/s wind speeds used in region A wind calculations so overturing should not be a problem with this structure in region A winds.

MODIFIED NODE

The capacity of the dome increases substantially with the following changes, (see Figure 20).

- Gusset welded to centre of ring
- Locking washes spaced further apart

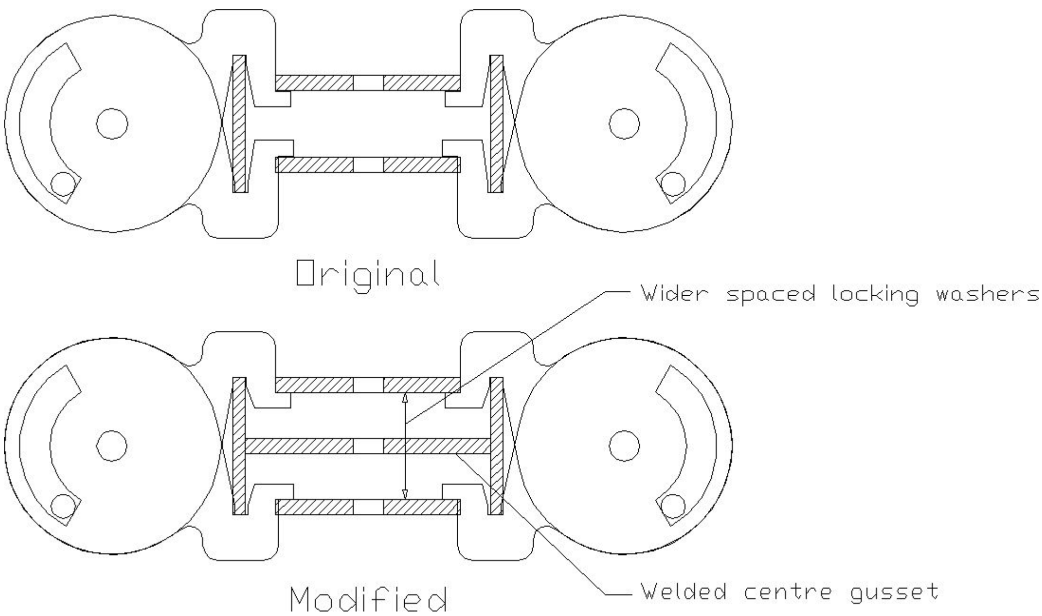


Figure 20: Differences between original and modified nodes

Finite Element Analysis

The modified node was compared to the original node using FE analysis. The results are shown in Figure 21. The modified node shows considerably less stress under a similar loading scenario raising the allowable strut force to around 9.0 kN.

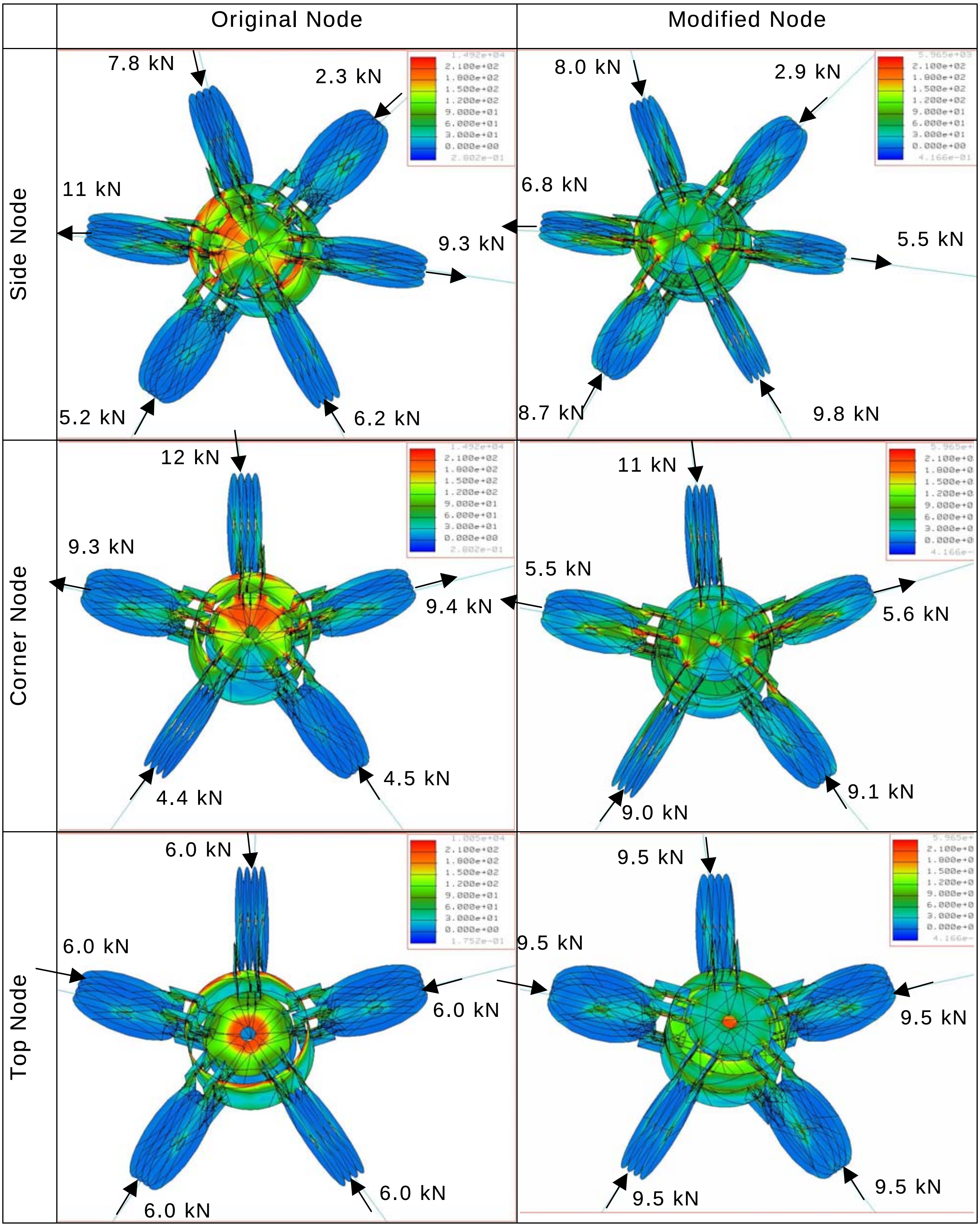


Figure 21: Comparison between strength of original and modifed nodes

Combined Loading

Table 13 shows the calculations used to determine the available strut capacity of the structure. It can be seen that the dome members have 8.1 kN of capacity to carry the safe working load.

Table 13: Structure loads

	Serviceability [kN]	Ultimate [kN]
Maximum allowable	9.0	20.0
Dead	-0.5 x 0.8 = -0.4	-0.5
Wind	-0.5	-0.9
Available Strut capacity	8.1	18.6

Safe Working Load

Table 14 summarises the safe working load of the structure so as not to exceed the capacity of the members determined in the previous section. It can be seen that 300 kg can be suspended from all nodes or 600 kg can be suspended from a single node.

Table 14: Safe working loads for modified node

Nodes	Safe Working Load [kg]
All	300
One	600

Figure 22 shows the maximum tension and compression force in the dome when 3,000 N is suspended from each node. It can be seen that the allowable force of 8.1 kN is not exceeded.

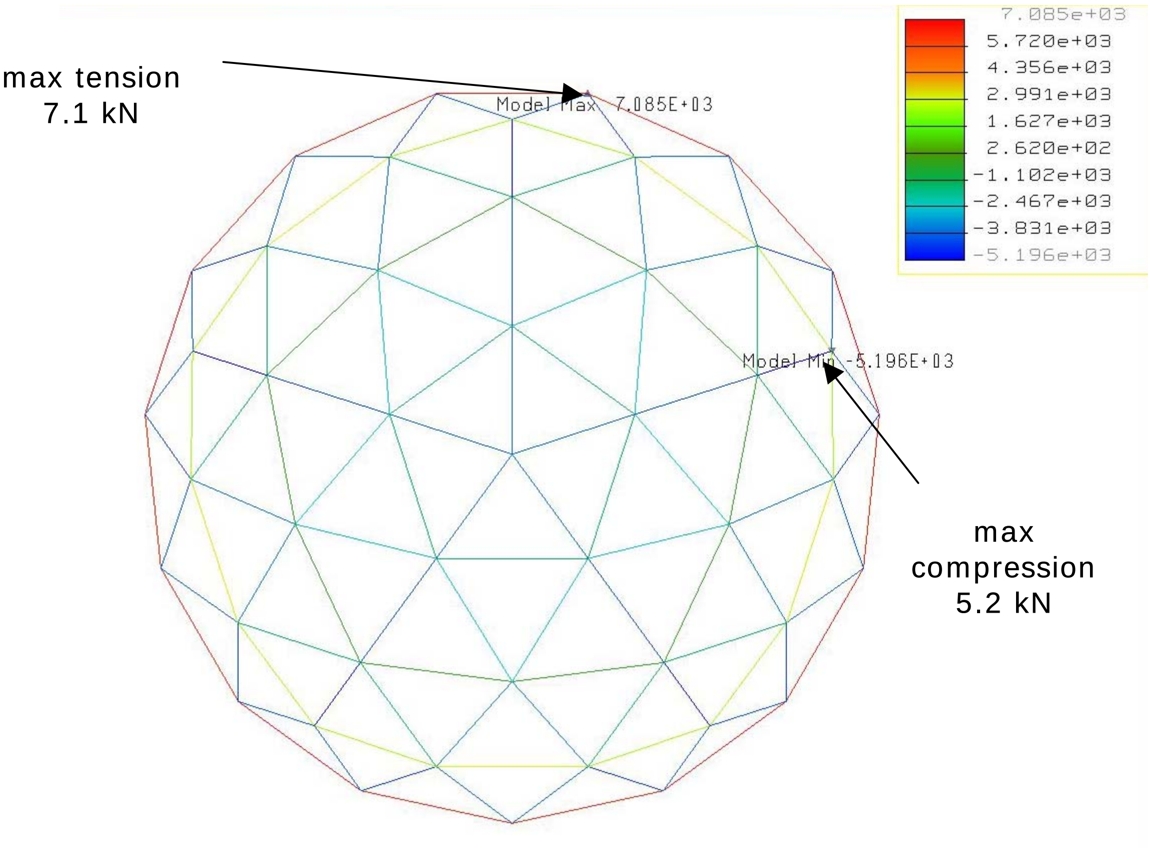


Figure 22: 3,000 N suspended from all nodes

Taking the worst case single node loading found in the previous load cases Figure 23 shows the affect on the structure of suspending a mass of 600 kg from a single node. It can be seen that the allowable force of 8.1 kN is not exceeded in any of the members.

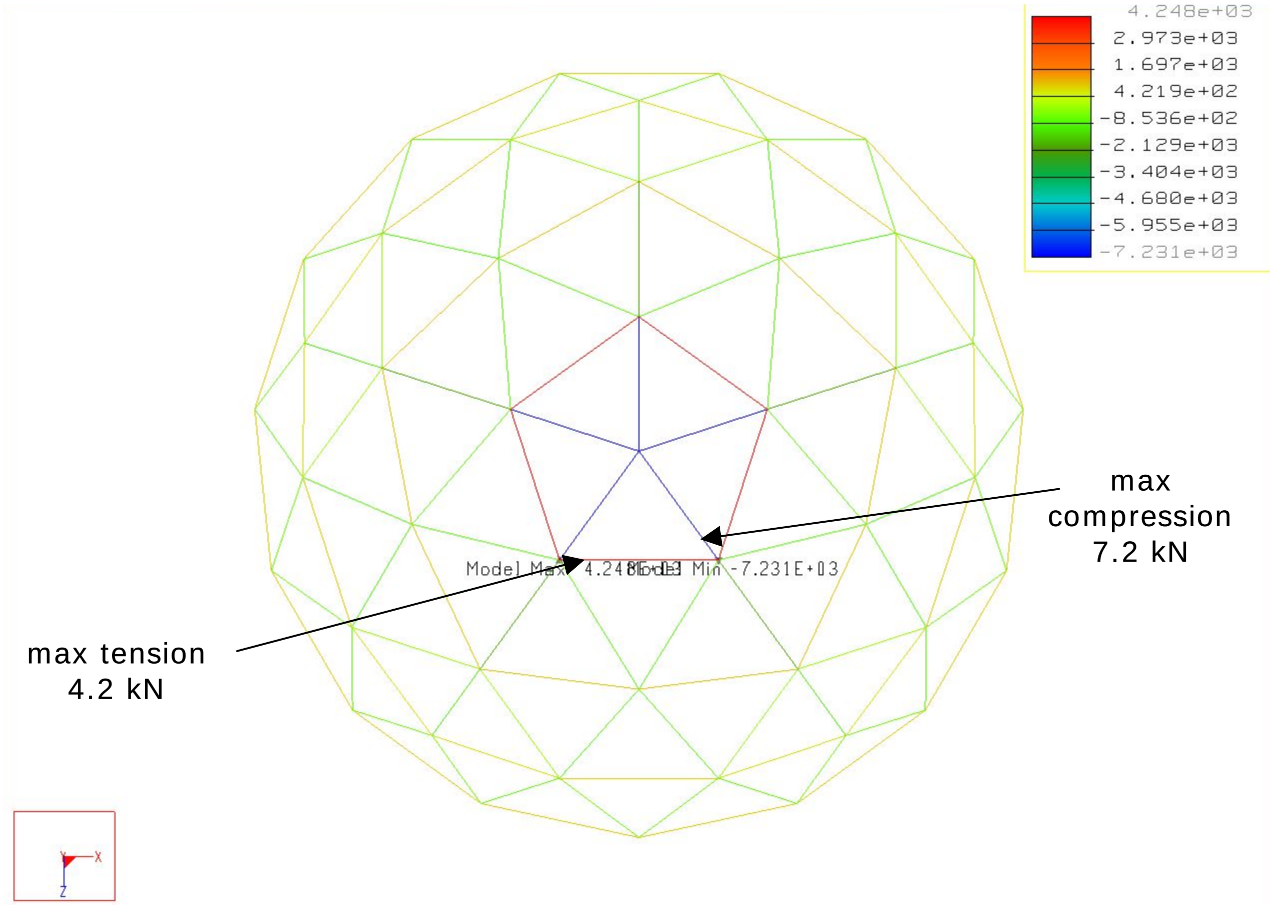


Figure 23: 6,000 N suspended from centre node (worst case single node loading)

SUSPENDED MASS FROM CENTRE OF BEAM

Horizontal members in the dome are typically in tension and therefore can carry a centre hanging load without concerns of weakening these members due to buckling. The allowable load to be suspended from the centre of a horizontal beam is given by:

$$\begin{aligned}
F &= \frac{0.6\sigma_y I}{yd} - F_{selfWeight} \\
&= \frac{0.6\sigma_y I}{y\frac{L}{2}} - L\rho g \\
&= \frac{0.6\times 350\times 10^{6}\times 42.9\times 10^{-9}}{0.025\times\frac{1.9}{2}} - (1.9\times 3.72\times 9.81) \\
&= 310\text{ N} \\
&\approx 30\text{ kg}
\end{aligned}$$

The safe working load to be suspended from the centre of any horizontal beam is 30 kg. This assumes the beam is not weakened in any way by the method used to attach the mass. For example this does not allow for holes to be drilled into beam to attach fixing points.

CONCLUSION

The 9 m Dome Dimensions dome has been analysed in accordance with the relevant sections of AS 1170: 2002 (Structural Design Action), AS 4100: 1998 (Steel Structures) and AISC for buckling of compact rolled shapes. Two versions of the dome have been considered depending on the node type. One node is the original design with no centre gusset and closely spaced locking washers; the other is the modified node with a welded centre gusset and widely spaced locking washers. The safe working loads for the dome with the two types of nodes are listed below in Table 15.

Table 15: Safe working loads

	Safe Working Load [kg]	
Nodes	Original Node Design	Modified Node Design
All	200	300
One	400	600
Centre beam mass (Horizontal beams only)	30	30

It is important to note that these safe working loads only apply when the locking washers used in the centre of the nodes are all in place as these are essential to the strength of the node.

The structure is suitable for region A winds which means it can be erected anywhere in Australia, excluding cyclone affected coastal areas during cyclonic conditions (Regions B, C and D of AS1170.2: 2002).